

# CoqHammer: Strong Automation for Program Verification

Łukasz Czajka

Cezary Kaliszyk

University of Copenhagen

University of Innsbruck

13 January 2018



# Interactive Proof in Type Theory

- Why do we love it?
  - The **power** we need
  - **Successful** projects
- Why do we hate it?
  - large parts of proofs are **tedious**

# Interactive Proof in Type Theory

- Why do we love it?
  - The **power** we need
  - **Successful** projects
- Why do we hate it?
  - large parts of proofs are **tedious**
- **Automation** for Interactive Proof
  - Tableaux: Itaut, Tauto, Blast
  - Rewriting: Simp, Subst, HORewrite
  - Decision Procedures: Congruence Closure, Ring, Omega, SMTCoq, ...

# Interactive Proof in Type Theory

- Why do we love it?
  - The **power** we need
  - **Successful** projects
- Why do we hate it?
  - large parts of proofs are **tedious**
- **Automation** for Interactive Proof
  - Tableaux: Itaut, Tauto, Blast
  - Rewriting: Simp, Subst, HORewrite
  - Decision Procedures: Congruence Closure, Ring, Omega, SMTCoq, ...
- **AI/ATP** techniques: Hammers
  - MizAR for Mizar
  - Sledgehammer for Isabelle/HOL
  - HOL(y)Hammer for HOL Light and HOL4
  - **CoqHammer** for Coq

# Hammers

- Hammer goal: provide efficient automated reasoning using facts from a large library.

# Hammers

- Hammer goal: provide efficient automated reasoning using facts from a large library.
- Strong relevance filtering.

# Hammers

- Hammer goal: provide efficient automated reasoning using facts from a large library.
- Strong relevance filtering.
- Usable library search “modulo simple reasoning”.



# Hammers

- Hammer goal: provide efficient automated reasoning using facts from a large library.
- Strong relevance filtering.
- Usable library search “modulo simple reasoning”.
  - We may not know the name of the lemma we want to apply.

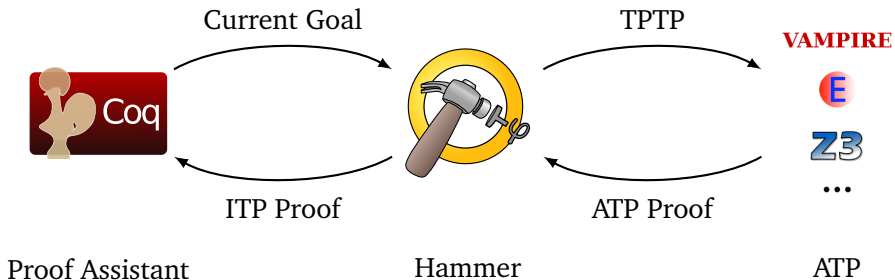
# Hammers

- Hammer goal: provide efficient automated reasoning using facts from a large library.
- Strong relevance filtering.
- Usable library search “modulo simple reasoning”.
  - We may not know the name of the lemma we want to apply.
  - There may be many equivalent formulations of the lemma – which one is used in the library?

# Hammers

- Hammer goal: provide efficient automated reasoning using facts from a large library.
- Strong relevance filtering.
- Usable library search “modulo simple reasoning”.
  - We may not know the name of the lemma we want to apply.
  - There may be many equivalent formulations of the lemma – which one is used in the library?
  - The exact lemma may not exist in the library, but it may “trivially” follow from a few other lemmas in the library.

# Hammer Overview



# Hammers

Hammers work in three phases.

# Hammers

Hammers work in three phases.

- Using machine-learning and AI techniques perform **premise-selection**: select about a few hundred to 1-2 thousand lemmas that are likely to be needed in the proof of the conjecture.

# Hammers

Hammers work in three phases.

- Using machine-learning and AI techniques perform **premise-selection**: select about a few hundred to 1-2 thousand lemmas that are likely to be needed in the proof of the conjecture.
- **Translate** the selected lemmas, together with the conjecture, from the logic of the ITP to a format accepted by powerful external automated theorem provers (ATPs) – most commonly untyped first-order logic with equality.

# Hammers

Hammers work in three phases.

- Using machine-learning and AI techniques perform **premise-selection**: select about a few hundred to 1-2 thousand lemmas that are likely to be needed in the proof of the conjecture.
- **Translate** the selected lemmas, together with the conjecture, from the logic of the ITP to a format accepted by powerful external automated theorem provers (ATPs) – most commonly untyped first-order logic with equality. Run the ATP(s) on the result of the translation.



# Hammers

Hammers work in three phases.

- Using machine-learning and AI techniques perform **premise-selection**: select about a few hundred to 1-2 thousand lemmas that are likely to be needed in the proof of the conjecture.
- **Translate** the selected lemmas, together with the conjecture, from the logic of the ITP to a format accepted by powerful external automated theorem provers (ATPs) – most commonly untyped first-order logic with equality. Run the ATP(s) on the result of the translation.
- **Reprove** the conjecture in the logic of the ITP, using the information obtained in the ATP runs.

# Hammers

Hammers work in three phases.

- Using machine-learning and AI techniques perform **premise-selection**: select about a few hundred to 1-2 thousand lemmas that are likely to be needed in the proof of the conjecture.
- **Translate** the selected lemmas, together with the conjecture, from the logic of the ITP to a format accepted by powerful external automated theorem provers (ATPs) – most commonly untyped first-order logic with equality. Run the ATP(s) on the result of the translation.
- **Reprove** the conjecture in the logic of the ITP, using the information obtained in the ATP runs. Typically, a list of (usually a few) lemmas needed by an ATP to prove the conjecture is obtained from an ATP run, and we try to reprove the goal from these lemmas.

# Evaluations

## Top-level goals:

- HOL(y)Hammer
  - Flyspeck text formalization: 47%
  - Similar results for HOL4
  - Slightly weaker for CakeML

# Evaluations

## Top-level goals:

- HOL(y)Hammer
  - Flyspeck text formalization: 47%
  - Similar results for HOL4
  - Slightly weaker for CakeML
- Sledgehammer
  - Probability theory: 40%
  - Term rewriting: 44%
  - Java threads: 59%

# Evaluations

## Top-level goals:

- HOL(y)Hammer
  - FLYspeck text formalization: 47%
  - Similar results for HOL4
  - Slightly weaker for CakeML
- Sledgehammer
  - Probability theory: 40%
  - Term rewriting: 44%
  - Java threads: 59%
- MizAR
  - Mizar Mathematical Library: 44%

# Evaluations

## Top-level goals:

- HOL(y)Hammer
  - FLYspeck text formalization: 47%
  - Similar results for HOL4
  - Slightly weaker for CakeML
- Sledgehammer
  - Probability theory: 40%
  - Term rewriting: 44%
  - Java threads: 59%
- MizAR
  - Mizar Mathematical Library: 44%
- **CoqHammer**
  - Coq standard library: 40%
  - Programming languages: ?

# CoqHammer demo

examples/imp.v

## CoqHammer: premise selection

- Learning done each time the plugin is invoked (to include *all* accessible facts).



## CoqHammer: premise selection

- Learning done each time the plugin is invoked (to include *all* accessible facts).
- Two machine-learning filters: k-NN and naive Bayes.

## CoqHammer: premise selection

- Learning done each time the plugin is invoked (to include *all* accessible facts).
- Two machine-learning filters: k-NN and naive Bayes.
- Re-uses the HOLyHammer efficient implementation (also adapted by Sledgehammer).

## CoqHammer: premise selection – features

- Features  $F(T)$  of theorem  $T$ 
  - Constants occurring in the statement (type) of  $T$ .
  - Constant-constant and constant-variable pairs that share an edge in the parse tree.

## CoqHammer: premise selection – features

```
Variable P : nat -> Prop.
```

```
T = forall k l, between k l -> k <= l.
```

## CoqHammer: premise selection – features

```
Variable P : nat -> Prop.
```

```
T = forall k l, between k l -> k <= l.
```

```
F(T) = { "Between.Between.between",  
         "Between.Between.between-X",  
         "Coq.Init.Datatypes.nat", "Coq.Init.Peano.le",  
         "Coq.Init.Peano.le-X" }
```

## CoqHammer: premise selection – dependencies

- Dependencies  $D(T)$  of theorem (resp. definition)  $T$ 
  - Constants occurring in the proof term (resp. unfolding) of  $T$ .

## CoqHammer: premise selection – dependencies

Proof term of  $T$ :

```
fun (k l : nat) (H : between P k l) =>
between_ind P k (fun l0 : nat => k <= l0) (le_n k)
  (fun (l0 : nat) (_ : between P k l0)
    (IHbetween : k <= l0) (_ : P l0) =>
      le_S k l0 IHbetween) l H
```

## CoqHammer: premise selection – dependencies

Proof term of  $T$ :

```
fun (k l : nat) (H : between P k l) =>
between_ind P k (fun l0 : nat => k <= l0) (le_n k)
  (fun (l0 : nat) (_ : between P k l0)
  (IHbetween : k <= l0) (_ : P l0) =>
  le_S k l0 IHbetween) l H
```

```
D(T) = { "Between.Between.between",
         "Between.Between.between_ind",
         "Coq.Init.Datatypes.nat", "Coq.Init.Peano.le",
         "Coq.Init.Peano.le_S", "Coq.Init.Peano.le_n",
         "P" }
```



## Translation: target logic

Target logic: untyped FOL with equality.

# Translation

Three functions  $\mathcal{F}$ ,  $\mathcal{G}$ , and  $\mathcal{C}$ .

- $\mathcal{F}$  : propositions  $\rightarrow$  FOL formulas  
used for  $\text{CIC}_0$  terms of type Prop.
- $\mathcal{G}$  : types  $\rightarrow$  guards  
used for  $\text{CIC}_0$  terms of type Type.
- $\mathcal{C}$  : all  $\text{CIC}_0 \rightarrow$  FOL terms

## Translation

- The function  $\mathcal{F}$  encodes propositions as FOL formulas and is used for terms of Coq having type Prop.

# Translation

- The function  $\mathcal{F}$  encodes propositions as FOL formulas and is used for terms of Coq having type Prop.
  - If  $\Gamma \vdash t : \text{Prop}$  then  $\mathcal{F}_\Gamma(\Pi x : t.s) = \mathcal{F}_\Gamma(t) \rightarrow \mathcal{F}_{\Gamma,x:t}(s)$ .
  - If  $\Gamma \not\vdash t : \text{Prop}$  then  $\mathcal{F}_\Gamma(\Pi x : t.s) = \forall x. \mathcal{G}_\Gamma(t, x) \rightarrow \mathcal{F}_{\Gamma,x:t}(s)$ .

# Translation

- The function  $\mathcal{F}$  encodes propositions as FOL formulas and is used for terms of Coq having type Prop.
  - If  $\Gamma \vdash t : \text{Prop}$  then  $\mathcal{F}_\Gamma(\Pi x : t.s) = \mathcal{F}_\Gamma(t) \rightarrow \mathcal{F}_{\Gamma,x:t}(s)$ .
  - If  $\Gamma \not\vdash t : \text{Prop}$  then  $\mathcal{F}_\Gamma(\Pi x : t.s) = \forall x.\mathcal{G}_\Gamma(t, x) \rightarrow \mathcal{F}_{\Gamma,x:t}(s)$ .
- The function  $\mathcal{G}$  encodes types as guards and is used for terms of Coq which have type Type.

# Translation

- The function  $\mathcal{F}$  encodes propositions as FOL formulas and is used for terms of Coq having type Prop.
  - If  $\Gamma \vdash t : \text{Prop}$  then  $\mathcal{F}_\Gamma(\Pi x : t.s) = \mathcal{F}_\Gamma(t) \rightarrow \mathcal{F}_{\Gamma,x:t}(s)$ .
  - If  $\Gamma \not\vdash t : \text{Prop}$  then  $\mathcal{F}_\Gamma(\Pi x : t.s) = \forall x.\mathcal{G}_\Gamma(t, x) \rightarrow \mathcal{F}_{\Gamma,x:t}(s)$ .
- The function  $\mathcal{G}$  encodes types as guards and is used for terms of Coq which have type Type.  
For instance, for a (closed) type  $\tau = \Pi x : \alpha.\beta(x)$  we have

$$\mathcal{G}(\tau, f) = \forall x.\mathcal{G}(\alpha, x) \rightarrow \mathcal{G}(\beta(x), f x)$$

# Translation

- The function  $\mathcal{F}$  encodes propositions as FOL formulas and is used for terms of Coq having type Prop.
  - If  $\Gamma \vdash t : \text{Prop}$  then  $\mathcal{F}_\Gamma(\Pi x : t.s) = \mathcal{F}_\Gamma(t) \rightarrow \mathcal{F}_{\Gamma,x:t}(s)$ .
  - If  $\Gamma \not\vdash t : \text{Prop}$  then  $\mathcal{F}_\Gamma(\Pi x : t.s) = \forall x.\mathcal{G}_\Gamma(t, x) \rightarrow \mathcal{F}_{\Gamma,x:t}(s)$ .
- The function  $\mathcal{G}$  encodes types as guards and is used for terms of Coq which have type Type.  
For instance, for a (closed) type  $\tau = \Pi x : \alpha.\beta(x)$  we have

$$\mathcal{G}(\tau, f) = \forall x.\mathcal{G}(\alpha, x) \rightarrow \mathcal{G}(\beta(x), f x)$$

- The function  $\mathcal{C}$  encodes Coq terms as FOL terms.

# Translation

- The function  $\mathcal{F}$  encodes propositions as FOL formulas and is used for terms of Coq having type Prop.
  - If  $\Gamma \vdash t : \text{Prop}$  then  $\mathcal{F}_\Gamma(\Pi x : t.s) = \mathcal{F}_\Gamma(t) \rightarrow \mathcal{F}_{\Gamma,x:t}(s)$ .
  - If  $\Gamma \not\vdash t : \text{Prop}$  then  $\mathcal{F}_\Gamma(\Pi x : t.s) = \forall x.\mathcal{G}_\Gamma(t, x) \rightarrow \mathcal{F}_{\Gamma,x:t}(s)$ .
- The function  $\mathcal{G}$  encodes types as guards and is used for terms of Coq which have type Type.

For instance, for a (closed) type  $\tau = \Pi x : \alpha.\beta(x)$  we have

$$\mathcal{G}(\tau, f) = \forall x.\mathcal{G}(\alpha, x) \rightarrow \mathcal{G}(\beta(x), f x)$$

- The function  $\mathcal{C}$  encodes Coq terms as FOL terms.
  - $\mathcal{C}_\Gamma(ts)$  is equal to:



# Translation

- The function  $\mathcal{F}$  encodes propositions as FOL formulas and is used for terms of Coq having type Prop.
  - If  $\Gamma \vdash t : \text{Prop}$  then  $\mathcal{F}_\Gamma(\Pi x : t.s) = \mathcal{F}_\Gamma(t) \rightarrow \mathcal{F}_{\Gamma,x:t}(s)$ .
  - If  $\Gamma \not\vdash t : \text{Prop}$  then  $\mathcal{F}_\Gamma(\Pi x : t.s) = \forall x.\mathcal{G}_\Gamma(t, x) \rightarrow \mathcal{F}_{\Gamma,x:t}(s)$ .
- The function  $\mathcal{G}$  encodes types as guards and is used for terms of Coq which have type Type.

For instance, for a (closed) type  $\tau = \Pi x : \alpha.\beta(x)$  we have

$$\mathcal{G}(\tau, f) = \forall x.\mathcal{G}(\alpha, x) \rightarrow \mathcal{G}(\beta(x), f x)$$

- The function  $\mathcal{C}$  encodes Coq terms as FOL terms.
  - $\mathcal{C}_\Gamma(ts)$  is equal to:
    - $\varepsilon$  if  $\Gamma \vdash ts : \alpha : \text{Prop}$ ,

# Translation

- The function  $\mathcal{F}$  encodes propositions as FOL formulas and is used for terms of Coq having type Prop.
  - If  $\Gamma \vdash t : \text{Prop}$  then  $\mathcal{F}_\Gamma(\Pi x : t.s) = \mathcal{F}_\Gamma(t) \rightarrow \mathcal{F}_{\Gamma,x:t}(s)$ .
  - If  $\Gamma \not\vdash t : \text{Prop}$  then  $\mathcal{F}_\Gamma(\Pi x : t.s) = \forall x.\mathcal{G}_\Gamma(t, x) \rightarrow \mathcal{F}_{\Gamma,x:t}(s)$ .
- The function  $\mathcal{G}$  encodes types as guards and is used for terms of Coq which have type Type.

For instance, for a (closed) type  $\tau = \Pi x : \alpha.\beta(x)$  we have

$$\mathcal{G}(\tau, f) = \forall x.\mathcal{G}(\alpha, x) \rightarrow \mathcal{G}(\beta(x), f x)$$

- The function  $\mathcal{C}$  encodes Coq terms as FOL terms.
  - $\mathcal{C}_\Gamma(ts)$  is equal to:
    - $\varepsilon$  if  $\Gamma \vdash ts : \alpha : \text{Prop}$ ,
    - $\mathcal{C}_\Gamma(t)$  if  $\Gamma \vdash s : \alpha : \text{Prop}$ ,

# Translation

- The function  $\mathcal{F}$  encodes propositions as FOL formulas and is used for terms of Coq having type Prop.
  - If  $\Gamma \vdash t : \text{Prop}$  then  $\mathcal{F}_\Gamma(\Pi x : t.s) = \mathcal{F}_\Gamma(t) \rightarrow \mathcal{F}_{\Gamma,x:t}(s)$ .
  - If  $\Gamma \not\vdash t : \text{Prop}$  then  $\mathcal{F}_\Gamma(\Pi x : t.s) = \forall x. \mathcal{G}_\Gamma(t, x) \rightarrow \mathcal{F}_{\Gamma,x:t}(s)$ .
- The function  $\mathcal{G}$  encodes types as guards and is used for terms of Coq which have type Type.

For instance, for a (closed) type  $\tau = \Pi x : \alpha. \beta(x)$  we have

$$\mathcal{G}(\tau, f) = \forall x. \mathcal{G}(\alpha, x) \rightarrow \mathcal{G}(\beta(x), f x)$$

- The function  $\mathcal{C}$  encodes Coq terms as FOL terms.
  - $\mathcal{C}_\Gamma(ts)$  is equal to:
    - $\varepsilon$  if  $\Gamma \vdash ts : \alpha : \text{Prop}$ ,
    - $\mathcal{C}_\Gamma(t)$  if  $\Gamma \vdash s : \alpha : \text{Prop}$ ,
    - $\mathcal{C}_\Gamma(t)\mathcal{C}_\Gamma(s)$  otherwise.

# Translation

- The function  $\mathcal{F}$  encodes propositions as FOL formulas and is used for terms of Coq having type Prop.
  - If  $\Gamma \vdash t : \text{Prop}$  then  $\mathcal{F}_\Gamma(\Pi x : t.s) = \mathcal{F}_\Gamma(t) \rightarrow \mathcal{F}_{\Gamma,x:t}(s)$ .
  - If  $\Gamma \not\vdash t : \text{Prop}$  then  $\mathcal{F}_\Gamma(\Pi x : t.s) = \forall x. \mathcal{G}_\Gamma(t, x) \rightarrow \mathcal{F}_{\Gamma,x:t}(s)$ .
- The function  $\mathcal{G}$  encodes types as guards and is used for terms of Coq which have type Type.

For instance, for a (closed) type  $\tau = \Pi x : \alpha. \beta(x)$  we have

$$\mathcal{G}(\tau, f) = \forall x. \mathcal{G}(\alpha, x) \rightarrow \mathcal{G}(\beta(x), f x)$$

- The function  $\mathcal{C}$  encodes Coq terms as FOL terms.
  - $\mathcal{C}_\Gamma(ts)$  is equal to:
    - $\varepsilon$  if  $\Gamma \vdash ts : \alpha : \text{Prop}$ ,
    - $\mathcal{C}_\Gamma(t)$  if  $\Gamma \vdash s : \alpha : \text{Prop}$ ,
    - $\mathcal{C}_\Gamma(t)\mathcal{C}_\Gamma(s)$  otherwise.
  - $\mathcal{C}_\Gamma(\lambda \vec{x} : \vec{t}.s) = F\vec{y}$  where  $s$  does not start with a lambda-abstraction any more,  $F$  is a fresh constant,  $\vec{y} = \text{FV}(\lambda \vec{x} : \vec{t}.s)$  and  $\forall \vec{y}. \mathcal{F}_\Gamma(\forall \vec{x} : \vec{t}. F\vec{y}\vec{x} = s)$  is a new axiom.

## Translation: translating inductive declarations

For inductive types:

- Translate the typing of each constructor (using the  $\mathcal{G}$  function).

## Translation: translating inductive declarations

For inductive types:

- Translate the typing of each constructor (using the  $\mathcal{G}$  function).
- Add axioms stating injectivity of constructors, axioms stating non-equality of different constructors, and the “inversion” axioms for elements of the inductive type.

## Translation: translating inductive declarations

For inductive types:

- Translate the typing of each constructor (using the  $\mathcal{G}$  function).
- Add axioms stating injectivity of constructors, axioms stating non-equality of different constructors, and the “inversion” axioms for elements of the inductive type.
- Translate the typing of the inductive definition.

## ATP invocation

- We use Vampire, E prover, and Z3.



## ATP invocation

- We use Vampire, E prover, and Z3.
- The provers may be run in parallel with different numbers of premises and premise selection methods.

# Proof reconstruction

- Use dependencies from a successful ATP run.

# Proof reconstruction

- Use dependencies from a successful ATP run.
- Do automatic proof search using different versions of our tactics (implemented in Ltac), with a fixed time limit for each.

# Proof reconstruction

- Use dependencies from a successful ATP run.
- Do automatic proof search using different versions of our tactics (implemented in Ltac), with a fixed time limit for each.
- 85.2% of proofs reconstructed.

# Proof search

$$\frac{\text{eauto}[\Gamma \vdash G]}{\Gamma \vdash G} \quad \frac{\text{congruence}[\Gamma \vdash G]}{\Gamma \vdash G} \quad \frac{\text{constructor}[\Gamma \vdash G]}{\Gamma \vdash G} \quad \frac{\text{easy}[\Gamma \vdash G]}{\Gamma \vdash G}$$

Leaf tactics (b)

$$\frac{\Gamma; A[?e_1/x_1, \dots, ?e_n/x_n] \vdash G \quad \Gamma; B[?e_1/x_1, \dots, ?e_n/x_n] \vdash G}{\Gamma; \forall x_1 \dots x_n, A \vee B \vdash G} \text{ (orsplit)} \quad \frac{\Gamma; A[?e_1/x_1, \dots, ?e_n/x_n] \vdash G \quad y \text{ fresh}}{\Gamma; \forall x_1 \dots x_n, \exists y, A \vdash G} \text{ (exsplit)}$$

Final splitting (e)

$$\frac{\text{destruct}(t)[\Gamma \vdash C[\text{match } t \text{ with } b]]}{\Gamma \vdash C[\text{match } t \text{ with } b]} \quad \frac{\text{destruct}(t)[\Gamma; C[\text{match } t \text{ with } b]] \vdash G}{\Gamma; C[\text{match } t \text{ with } b] \vdash G} \quad \frac{\Gamma \vdash G_1 \quad \Gamma \vdash G_2}{\Gamma \vdash G_1 \wedge G_2}$$

Splitting (e)

$$\frac{\Gamma; A; B \vdash G}{\Gamma; A; A \rightarrow B \vdash G} \quad \frac{\Gamma; \forall x_1 \dots x_n, A; \forall x_1 \dots x_n, B \vdash G}{\Gamma; \forall x_1 \dots x_n, A \wedge B \vdash G}$$

$$\frac{\Gamma; \forall x_1 \dots x_n, A \rightarrow B \rightarrow C \vdash G}{\Gamma; \forall x_1 \dots x_n, A \wedge B \rightarrow C \vdash G} \quad \frac{\Gamma; \forall x_1 \dots x_n, A \rightarrow C; \forall x_1 \dots x_n, B \rightarrow C \vdash G}{\Gamma; \forall x_1 \dots x_n, A \vee B \rightarrow C \vdash G} \quad \frac{\Gamma; A \vdash G \quad x \text{ fresh}}{\Gamma; \exists x, A \vdash G} \quad \frac{\Gamma; A \vdash G}{\Gamma; A; A \vdash G}$$

Hypothesis simplification (e)

$$\frac{\Gamma; x : A \vdash B}{\Gamma \vdash \forall x : A, B} \text{ (intro)}$$

Introduction (e)

$$\frac{}{\Gamma; \text{False} \vdash G} \quad \frac{}{\Gamma; G \vdash G} \quad \frac{\Gamma \vdash G[?e/x]}{\Gamma \vdash \exists x, G} \quad \frac{\Gamma \vdash G_1 \quad \Gamma \vdash G_2}{\Gamma \vdash G_1 \vee G_2} \quad \frac{\Gamma \vdash G_2}{\Gamma \vdash G_1 \vee G_2}$$

$$\frac{\text{yapply}(H)[\Gamma; H : A \vdash P]}{\Gamma; H : A \vdash P} \text{ (apply)} \quad \frac{\text{yrewrite}(H)[\Gamma; H : \forall x_1 \dots x_n, A = B \vdash G]}{\Gamma; H : \forall x_1 \dots x_n, A = B \vdash G} \text{ (rewrite)}$$

$$\frac{\Gamma; \forall x_{k+1} \dots x_n, \exists x, A[?e_1/x_1, \dots, ?e_k/x_k] \vdash G}{\Gamma; \forall x_1 \dots x_k \dots x_n, \exists x, A \vdash G} \text{ (exinst)} \quad \frac{\Gamma; T'_{j_1}, \dots, T'_{j_m}, A'_1 \vdash G \quad \Gamma; T'_{j_1}, \dots, T'_{j_m}, A'_2 \vdash G \quad \Gamma; T'_{j_1}, \dots, T'_{j_{m-k-1}} \vdash T'_k}{\Gamma; \forall (x_1 : T_1) \dots (x_n : T_n), A_1 \vee A_2 \vdash G} \text{ (orinst)}$$

Proof search (b)

$$\frac{\Gamma; \forall x_1 \dots x_n, \text{False} \vdash \text{False}}{\Gamma; \forall x_1 \dots x_n, \text{False} \vdash G} \quad \frac{\text{inversion}(H)[\Gamma; H : P \vdash G]}{\Gamma; H : P \vdash G} \text{ (invert)} \quad \frac{\text{destruct}(x)[\Gamma; x : T \vdash G]}{\Gamma; x : T \vdash G} \text{ (destruct)}$$

Initial proof search (b)

# Overall hammer evaluation

All statements from the Coq standard library

ATP success **50%**

- ATPs used: E, Z3, Vampire with 30 seconds time limit

Overall success **40.8%**

- 8 threads with different lemma selection, premises, provers, reconstruction

Works well for program semantics formalizations.

# Conclusion

- CoqHammer optimized for programming language formalization.

# Conclusion

- CoqHammer optimized for programming language formalization.
- Proof length already close to that of Isabelle/HOL.



# Conclusion

- CoqHammer optimized for programming language formalization.
- Proof length already close to that of Isabelle/HOL.
- Improvements needed for dependent types and boolean reflection.