**Exercise 1** (The Curry-Howard isomorphism).

Which of the following formulas are intuitionistically provable? Either give a proof by explicitly providing an appropriate Coq term or argue informally using the BHK interpretation that the formula is not intuitionistically provable. Do not use tactics. Which of the formulas are classically valid?

1. $(P \to Q) \to \neg Q \to \neg P$.

   *Hint.* Recall that $\neg P$ is defined as $P \to \bot$.
2. $(P \to P \to Q) \to P \to Q$.
3. $(P \to \neg P) \to \neg P$.
4. $(\neg P \to P) \to P$.
5. $P \to (P \to Q) \to Q$.
6. $P \to \neg\neg P$.
7. $\neg\neg P \to P$.
8. $P \vee \neg P \to \neg\neg P \to P$.

   *Hint.* Recall that one can prove an arbitrary formula by matching on a proof of $\bot$.
9. $\neg(P \vee Q) \leftrightarrow \neg P \wedge \neg Q$.
10. $\neg(P \wedge Q) \to \neg P \vee \neg Q$.
11. $\neg P \vee \neg Q \to \neg(P \wedge Q)$.
12. $\neg P \vee \neg\neg P$.
13. $(P \vee Q \to R) \to (P \to R) \wedge (Q \to R)$.
14. $\forall x \exists y R x y \to \exists y \forall x R x y$.
15. $\exists x(P \wedge R x) \leftrightarrow P \wedge \exists x R x$.
16. $\forall x(P \vee R x) \to P \vee \forall x R x$.
17. $P \vee \forall x R x \to \forall x(P \vee R x)$.
18. $\exists x R x \to \neg \forall x \neg R x$.
19. $\neg \forall x \neg R x \to \exists x R x$.
20. $\neg\neg \forall x R x \to \forall x \neg\neg R x$.