Ackermann's function

```
ackermann 0 n = n + 1
ackermann (m + 1) 0 = ackermann m 1
ackermann (m + 1) (n + 1) = ackermann m (ackermann (m + 1) n)
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Theorem

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ackermann (m + 1) n = iterate (ackermann m) (n + 1) 1
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Proof.

By

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Proof.

By induction on *n*.

- **Base case.** We need to show

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By induction on *n*.

- **Base case.** We need to show
  `ackermann (m + 1) 0 = iterate (ackermann m) (0 + 1) 1`.
  But this follows

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Proof.

By induction on n.

- **Base case.** We need to show `ackermann (m + 1) 0 = iterate (ackermann m) (0 + 1) 1`. But this follows from definitions by computation.

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Theorem

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Proof.

Inductive step. We need to show

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ackermann (m + 1) n = iterate (ackermann m) (n + 1) 1
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Proof.

Inductive step. We need to show (*):

```
ackermann (m + 1) (n + 1) = iterate (ackermann m) (n + 2) 1.
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The induction hypothesis is

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Now we use the inductive hypothesis, concluding:

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ackermann (m + 1) (n + 1) =
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Then our claim (*) follows

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Then our claim (*) follows from definitions by computation. □